

Determine the local extrema using the second derivative test

A) $y = x^2$

B) $y = -x^2$

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25) $f(x) = x^3 - 12x^2 + 45x$

$$f''(x) = 6x - 24$$

$$f'(x) = 3x^2 - 24x + 45$$

$$f''(3) = -6 < 0$$

$$0 = \frac{3x^2 - 24x + 45}{3}$$

$$0 = x^2 - 8x + 15$$

$$0 = (x-5)(x-3)$$

$$\text{C.P. } x=5, x=3$$

At $x=3$ $f(x)$ is concave down
so $x=3$ Local Max

$f''(5) = 6 > 0$
At $x=5$ $f(x)$ is concave up so $x=5$ is local min

27) $f(x) = 3x^4 - 8x^3 + 6x^2$

2nd derivative
test for local
extrema

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31) $f(x) = 6x^{3/2} - 4x^{1/2}$ $\rightarrow x \geq 0$

$$f(x) = 6x^{3/2} - 4x^{1/2}$$

$$f'(x) = 9x^{1/2} - 2x^{-1/2} \rightarrow f''(x) = \frac{9}{2}x^{-1/2} + x^{-3/2}$$

$$(\sqrt{x}) \quad 0 = 9\sqrt{x} - \frac{2}{\sqrt{x}}$$

$$f''(x) = \frac{9}{2\sqrt{x}} + \frac{1}{\sqrt{x^3}}$$

$f''(0) = \text{UND}$ 2nd derivative test for local extrema fails

$$0 = 9x - 2$$

$$x = \frac{2}{9} \text{ C.P.}$$

$$f''\left(\frac{2}{9}\right) > 0 \quad f(x) \text{ concave up}$$

$$x = \frac{2}{9} \text{ local min b/c } f'\left(\frac{2}{9}\right) = 0 \text{ and } f''\left(\frac{2}{9}\right) > 0$$

5) $f(x) = 10x^3 - x^5$

$f'(x)$ und
C.P. $x=0$

What you'll Learn About
 How to describe the key features of a graph using the 1st and 2nd derivative

2) $f(x) = -2x^3 + 6x^2 - 3$ $[-1, 3]$

$f'(x) = -6x^2 + 12x$

$f''(x) = -12x + 12$

$0 = -6x^2 + 12x$

$0 = -12x + 12$

$0 = -6x(x - 2)$

$x = 1$

C.P. $x = 0$ $x = 2$

$f''(0) = 12 > 0$

$(-1, 0)$ $f'(-1) = -18 < 0$ $f(x)$ dec

$x = 0$ local min b/c $f'(0) = 0$
 $f(x)$ = concave up $(-1, 1)$

$(0, 2)$ $f'(1) = 6 > 0$ $f(x)$ inc

$f''(2) = -12 < 0$

$(2, 3)$ $f'(3) = -18 < 0$ $f(x)$ dec

$x = 2$ local max b/c $f'(2) = 0$
 $f(x)$ concave down $(1, 3)$

Extreme Values: $f(x) = -2x^3 + 6x^2 - 3$

Abs Max $(-1, 5)$ and $(2, 5)$

$f(-1) = 2 + 6 - 3 = 5$

Abs Min $(0, -3)$ and $(3, -3)$

$f(0) = -3$

$f(2) = -16 + 24 - 3 = 5$

$f(3) = -54 + 54 - 3 = -3$